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Control of Nonlinear Structural Dynamic Systems: Chaotic Vibrations

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In recent years, chaotic behavior has been observed in a large number of physical systems. The appearance of these unpredictable, broadband frequency responses in structural dynamic systems presents a possible threat to the integrity of aerospace structures because of their erratic time histories. Thus, the objective of this paper is to investigate the control of chaotic motions through the simple example of a buckled beam. As an initial attempt, a nonlinear control methodology termed feedback linearization was utilized. In this method, the nonlinearities causing chaos are eliminated from the closed-loop system through nonlinear feedback. Although effective in eliminating chaotic motions, this scheme does have some disadvantages such as the demand for accurate models and, in general, full state measurements. As an alternative, the feasibility of using smart structural concepts to control chaotic vibrations was examined. This technique utilized the introduction of so-called electronic damping to move the system out of a chaotic parameter region. The method was shown to be feasible and holds promise for future applications. Thus, this work demonstrates that a controller capable of controlling a chaotic system can be developed. From this study, directions for further research in the area of controlling chaotic systems were obtained.

Nomenclature

A = cross-sectional area
c = linear viscous damping constant
E = elastic modulus
f = distributed time-dependent load
I = moment of inertia of cross section
K = membrane stiffness parameter
l = length of the beam
N = averaged tensile load in the beam
P = constant applied axial load
r = radius of gyration, $\sqrt{I/A}$
t = time
w = transverse displacement of beam
x = coordinate along the beam
ρ = density

Introduction

IT is well known that all physical systems exhibit nonlinear behavior, yet because of the complexities associated with nonlinear analysis, engineers have traditionally attempted to control systems using linear theories. Recently, due to great advances in computer technology, nonlinear control techniques have gained importance, but their applications are still limited to certain classes of nonlinear behavior. One very interesting class of nonlinear behavior that has yet to receive a lot of attention in the control field is chaos. In the field of nonlinear dynamics, chaotic systems have received considerable attention in the literature over the past two decades. Primarily, studies have focused on detecting and characterizing these strange vibrations, understanding their complex mathematical structure, and predicting thresholds at which chaos first appears. References 1-7 provide a brief sample of the many papers written on the subject. Being a relatively new phenomenon, however, little work has been completed on why chaos is important or the effects chaotic vibrations may have on the integrity of an aerospace structure. It is conceivable, however, that the broadband spectrum of a chaotic vibration could lead to a change in the crack growth pattern in a given structure. In this manner, chaotic vibrations could actually harm a system. Chaotic vibrations also tend to appear as a system undergoes a transition from one form of deformation to another, such as in the buckled-beam problem[3] where chaos occurs as the beam changes from motion about one equilibrium point to motion about three equilibrium points. Thus, chaos could foreshadow a motion that would degrade the integrity of a system even if the vibrations themselves cause no harm. In either of these cases, it is anticipated that chaotic regions should be avoided. In many situations, however, it is not possible to simply avoid the nonlinear response of a system through design specifications and an appropriate structural design. Therefore, it is necessary to develop active controllers capable of either displacing a structure from a chaotic regime or preventing chaos from occurring. The development of such a controller is the focus of this paper.

There have been studies of the chaotic nature of some feedback control systems,[8,9] but the authors are aware of very few studies with the objective of eliminating chaotic motions. Recently, Hübler et al.[10,11] addressed the concept of controlling chaotic systems through studies of nonlinear maps. The objective of the work reported in Refs. 10 and 11 was to force a map to simulate a specified response. However, the work has not addressed applications to ordinary differential equation (ODE) models of vibrating systems, nor has it involved any practical implementations. Therefore, this paper will address these issues in general and then demonstrate the applications of controller designs, feedback linearization, and smart structural concepts through the study of a simple ODE beam model, which includes a bending-stretching coupling term that induces chaotic motions for some forcing amplitudes.

Our present interest in controlling chaotic, or more generally nonlinear, vibrations has evolved from work focusing on the use of piezoceramic sensors and actuators to control linear vibrations in Euler beams. It is recognized, however, that many existing control methodologies may be applicable to the present problem of controlling chaos because, in general, chaos can be considered to be simply a subset of a larger class of nonlinear phenomenon. One such control scheme, feedback linearization, will be examined in this paper. The theory be-

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hind this scheme has been described by several authors (see, for example, Isidori[12] or Slotine and Li[13]) and has been practically applied to systems such as helicopters and robots. Simply stated, this method utilizes a control force that includes a term that exactly cancels any nonlinearities in the plant, resulting in a linear closed-loop system. For the control of chaos, this method is particularly applicable since in the absence of any nonlinearities, chaos cannot occur. If stability problems develop due to the elimination of nonlinearities, a second control term may be chosen using existing linear control theory to obtain any desired behavior. Feedback linearization, however, has some disadvantages in practical applications such as the requirement, in most systems, that all states be available for feedback. In addition, methods of compensating for inaccuracies in the system model must be developed. Thus, the use of piezoceramic sensors and actuators coupled by simple rate feedback is also considered in this paper.

The use of integrated piezoceramic sensors and actuators, a so-called smart structural concept, may also be useful for controlling chaos because of the fact that damping can determine whether or not a system vibrates chaotically. It has been well documented that piezoceramic sensors and actuators, coupled with rate feedback, can significantly change the damping of a linear system. The use of such a control scheme to damp out specific modes in the response of a linear structural system has been previously documented by authors.[14-16] In this paper, several analytical studies demonstrate the feasibility of using such controllers to control the nature of the response of a nonlinear structural dynamic system.

For demonstration of the control schemes, the example of a simply supported, buckled beam will be used in the present study. A model of this system will be given, followed by a general discussion of the requirements on a controller for a chaotic system. Next, the feedback linearization technique will be discussed and applied to the buckled-beam example. Finally, the feasibility of the piezoceramic control scheme will be examined through the buckled-beam model, and the numerical results compared to those of the feedback linearization scheme.

Single-Input, Single-Output System

To demonstrate the use of the control schemes to be developed, a simple ODE model of a simply supported Euler-Bernoulli beam will be utilized. This model has been examined by many researchers[1,2,3,7] and is an excellent example of a chaotic system. For a given set of parameter values, a sinusoidal forcing function of small amplitude causes the beam to oscillate about one of its buckled equilibrium points. As the load amplitude is increased, however, the beam goes through a period-doubling cascade, culminating in a region of chaotic behavior. For larger amplitudes, the beam returns to a periodic motion, but now the motion occurs about all three of the beams' equilibrium positions. Thus, chaos serves as a transition between two distinct types of motion.

The ODE model of the beam is found using a Galerkin approximation to the well-known PDE model of an Euler-Bernoulli beam with axial coupling

$$\rho A \frac{\partial^2 w}{\partial t^2} + c\frac{\partial w}{\partial t} + EI\frac{\partial^4 w}{\partial x^4} - N\frac{\partial^2 w}{\partial x^2} = f \tag{1}$$

where a viscous damping term has been added to the model. The total axial force N consists of the effects from an applied axial load P and membrane stretching. If we assume that the beam has simple supports with axial constraints at both boundaries, N can be expressed as

$$N = P + \frac{EA}{2l}\int_0^l \left(\frac{\partial w}{\partial x}\right)^2 \mathrm{d}x$$

This beam model was one of the first structural systems to be studied for its chaotic motions in a paper by Holmes.[3] Equation (1) can be nondimensionalized using the following nondimensional parameters:

$$\tau = \frac{t}{l^2}\sqrt{\frac{EI}{\rho A}}, \qquad \xi = \frac{x}{l}$$

$$\phi = \frac{w}{r}, \qquad r = \sqrt{\frac{I}{A}}$$

$$\Gamma = \frac{PL^2}{EI}, \qquad \tilde{N} = \frac{NL^2}{EI}$$

$$\zeta = \frac{cl^2}{\sqrt{\rho AEI}}, \qquad F = \frac{fL^4}{EIr}$$

which yields

$$\ddot{\phi} + \zeta\dot{\phi} + \phi'''' - \tilde{N}\phi'' = F \tag{2}$$

with

$$\tilde{N} = \Gamma + K\int_0^1 (\phi')^2\,\mathrm{d}\xi$$

In these expressions, overdots represent partial differentiation with respect to nondimensional time τ, and primes partial differentiation with respect to nondimensional position ξ. K denotes a membrane-stretching parameter and for the given nondimensionalization has a value of one-half. Also note that for a compressive load, Γ will be negative.

Next, the PDE is reduced to an ODE using Galerkin's method, which will provide the accuracy adequate for the present study. To perform the reduction, it is first assumed that, in general, the solution of the PDE can be written approximately as

$$\phi(\xi,\tau) = \sum_{n=1}^{M} q_n(t)\varphi_n(x)$$

where q_n represents a time-dependent amplitude and φ_n a spatial basis function, typically the linear modes of the beam. For the present study, the objective is to develop control laws for a single-input, single-output (SISO) system. Thus, a forcing function of the form

$$F = F_0 \sin \pi\xi \sin \omega\tau$$

will be assumed. This loading causes the beam to vibrate primarily in its first mode in the steady state, and thus a single mode accurately models the system. Experimentally, such a forcing function would be difficult to obtain, but discrete approximations could be obtained with arrays of actuators.

For the simply supported beam, choose the single mode as $\varphi = \sin \pi\xi$, which satisfies all the given boundary conditions and results in a zero boundary residual term. Then the Galerkin procedure yields

$$\ddot{q} + \zeta\dot{q} - \beta q + \alpha q^3 = F_0 \sin \omega\tau \tag{3}$$

where

$$\beta = (\Gamma\pi^2 - \pi^4)$$

$$\alpha = \frac{K\pi^4}{2} = \frac{\pi^4}{4}$$

If we assume that the controller exerts a force on the beam and the system output is the velocity at the midpoint of the beam, this model can be expressed as a SISO system

$$\dot{x}_1 = x_2 \tag{4}$$

$$\dot{x}_2 = -\zeta x_2 + \beta x_1 - \alpha x_1^3 + F_0 \sin \omega\tau \sin \pi\xi + U + E \tag{5}$$

$$y = x_2 \tag{6}$$

where $x_1 = q$, $x_2 = \dot{q}$, y is the output, U is the control force (input), and E is any noise that may enter the system.

Detection of Chaos

To develop a control scheme for chaotic systems, it is necessary to first define the objectives of the controller. Simply stated, this objective is to detect the onset of chaos and then act by eliminating or avoiding any chaotic responses. Notice that the goal is not to track a certain reference exactly, just to cause the system to exhibit a certain class of behavior, that of a periodic response. As will be shown next, the elimination of chaotic behavior can be readily accomplished using either feedback linearization or smart structures. However, the problem of determining, at a given instant in time, whether or not a system is oscillating chaotically is more difficult. To start, much of the theory of chaotic systems is related to ergodic theory and thus long time averages are required to precisely define chaos. However, if the characteristics of a given system are known in advance, possibly from a study of an analytical model, it may be possible to anticipate when chaos might occur. For example, in the buckled-beam problem, if the forcing function is initially small and gradually increases with time, it will be possible to detect a change from periodic behavior. In this manner, even though it is not possible to prove that a system is vibrating chaotically, it is possible to anticipate its onset and act accordingly. To accomplish this, the theory behind a controller can be expected to be based on Poincaré sections of the response of the system.

From analytical studies, such as that of Hall and Hanagud,[17] it has been noted that as a system goes through the transition from periodic to chaotic motion, the bound of the set of points comprising a Poincaré section of a trajectory increases. Here, the term bound is used to mean the size of a box in the phase space that contains all points in the Poincaré section. For example, the single point in the Poincaré section of a single harmonic trajectory is bounded by an infinitesimally small square, whereas the two points of a Poincaré section of a period-doubled motion are bounded by a rectangle with a diagonal equal to the length between the two points. As bifurcations occur, this bound grows until a limit is reached where a strange attractor with a nonintegral fractal dimension appears, indicating chaos. For the present study, this growth can be utilized to predict the onset of chaos. It is possible to define a bounded region in the phase space of the system such that if a point in the Poincaré section leaves this region, the motion of the system is near-chaotic. If bifurcation to another periodic motion is possible, a check to determine if subsequent points remain bounded about a new nominal point could also be included. This type of knowledge, as well as a feel for approximate bounds that indicate the onset of chaotic behavior, can be attained through the study of analytical models.

Practically speaking, the foregoing theory translates into a discrete sampling of the output sensor signals, a possible integration of this signal, and a simple algorithm comparing the data to established bounds about nominal positions. For direct analogy to a Poincaré map, the sample would be connected to the forcing so that the sampling time could be equated to the period of the forcing function. Once a point from the sampling violates the prescribed bounds, the controller would activate actuators. Depending on the specific problem, an algorithm would also be required to decide if and when the actuators would be deactivated. A block diagram of the logic required for such a controller is given in Fig. 1.

Feedback Linearization

With the objectives of the controller defined, the construction of a control system can be addressed. Begin with the fact that chaos cannot develop in a linear system. Thus, if a given chaotic system can be made to simulate the response of a linear system, chaos would not develop. Feedback linearization is a control technique that will perform this transformation. For a general single-input, single-output nonlinear system, the basic idea is to transform the system into an equivalent linear system through the use of a state transformation $z = T(x)$ and an appropriate control input $\tilde{U} = \tilde{U}(x, \tilde{u})$. In this form, the equivalent linear control force $\tilde{u}$ can be determined from traditional linear methods to yield a desired behavior. Both Isidori[12] and Slotine and Li[13] describe a technique termed input-output linearization that will be demonstrated here using the SISO system from the buckled-beam model.

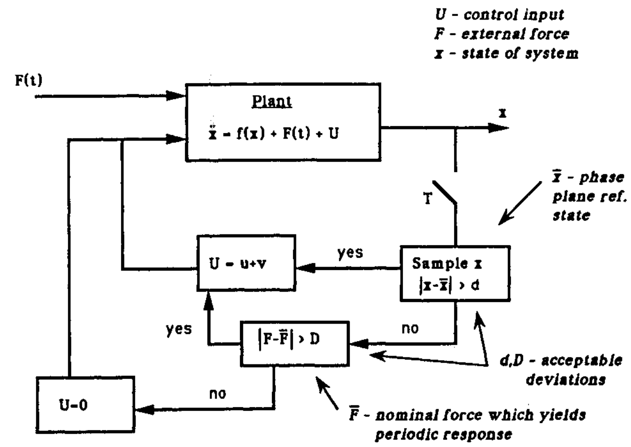

Fig. 1 Controller logic.

First, it is assumed that all states are available for feedback, but that there is only a single output. For the beam model, velocity will be assumed to be the output. Consider then a general system in state-space form

$$[\dot{x}] = \left[f(x) + g(x)\tilde{U}\right] \tag{7}$$

$$y = h(x) \tag{8}$$

where for the buckled-beam model [Eqs. (4–6)]

$$x = [x_1 \quad x_2]^T \tag{9}$$

$$f(x) = [x_2 \quad -\zeta x_2 + \beta x_1 - \alpha x_1^3]^T \tag{10}$$

$$g(x) = [0 \quad 1]^T \tag{11}$$

$$h(x) = x_2 \tag{12}$$

and $\tilde{U}$ is a combination of the control force and any external loads or disturbances

$$\tilde{U} = U + E + F_0 \sin \omega\tau \sin \pi\xi \tag{13}$$

The initial step in the linearization process is to determine the relative degree of the system. Generally speaking, it is the number of times the output must be differentiated to obtain a relation in which the input appears explicitly.[12] Formally, the relative degree is found by examining appropriate Lie derivatives of the output function $L_g L_f^k h$, where the relative degree r is defined as $r = \bar{k} + 1$, where $\bar{k}$ is the value of k for which the given derivative is nonzero and all smaller values of k make the derivative zero. For the buckled-beam problem,

$$L_g L_f^0 h = L_g h = \nabla h g = [01][01]^T = 1$$

which is not zero, so the relative degree is $r = 0 + 1 = 1$. Thus in this system, the relative degree is less than the order of the system. In such a situation, the feedback linearization technique requires that the first r-transformed coordinates be chosen as $z_i = L_f^i h$, $i = 0, r - 1$, while the remaining coordinates are chosen such that z has a nonsingular Jacobian and

$L_g z_i = 0$, $i = r, n-1$, where n is the order of the system. Thus for the buckled beam, choose the transformed state variables as

$$z_1 = h(x) = x_2$$

$$z_2 = x_1$$

which gives the transformed state equations in the form

$$\dot{z}_1 = \zeta z_1 + \beta z_2 - \alpha z_2^3 + \tilde{U} \tag{14}$$

$$\dot{z}_2 = z_1 \tag{15}$$

Note that the second equation describes only internal dynamics, but that the internal dynamics will not affect the stability of the system since they are dependent only on the output of the system.

For a system that has been transformed by the foregoing procedure, the linearization process is completed by application of the control law

$$\tilde{U} = \frac{1}{L_g L_f^{r-1} h} (-L_f^r h + \tilde{u})$$

which for the buckled-beam example becomes

$$\tilde{U} = \frac{1}{1} (\zeta x_2 - \beta x_1 + \alpha x_1^3 + \tilde{u})$$

where $\tilde{u}$ will be an equivalent linear control force summed with any system disturbances. Thus for the present example,

$$\tilde{u} = u + E + F_0 \sin \omega\tau$$

which yields a closed-loop system model of the form

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} u + E + F_0 \sin \omega\tau \\ z_1 \end{bmatrix}$$

Now, u can be selected to achieve any desired behavior using traditional linear techniques. A block diagram of the transformed system is given in Fig. 2. For practical implementation, a controller would develop $\tilde{U}$ from sensor inputs and send this signal to an actuator to close the loop. Because of the nonlinear dependence on deformation, however, the signal could quickly saturate the actuator and the system would no longer be linearized. Problems in the linearization algorithm could also occur if the system model had inaccuracies and some of the nonlinear effects were not removed. Thus, there are limitations to this approach.

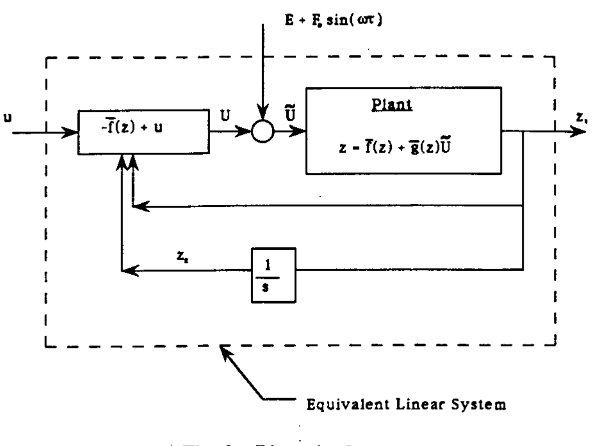

Fig. 2 Linearized system.

Numerical Studies

To demonstrate the effectiveness of the feedback linearization technique, the simple case of a proportional-plus-derivative control was used for the equivalent linear control. For this example, it is assumed that there is no noise in the system. If noise is present, existing methods for eliminating its effects can be used, such as that outlined in Ref. 15. To begin, assume the equivalent linear control force to be

$$u = -Kz_1 - Gz_2$$

where K and G are constant gains. Then in terms of the original variable q, the equation of motion of the closed-loop system becomes

$$\ddot{q} + K\dot{q} + Gq = F_0 \sin \omega\tau \tag{16}$$

This is a controllable, linear system in which the constant gains can be chosen to yield a stable system.

For a test of the effectiveness of the feedback linearization scheme and the controller, three cases were studied. For the test, gains of $K = \zeta$ and $G = \beta/2$ were selected to provide a stable equivalent linear system, where ζ and β are previously given system parameters. In all studies, the following parameter values were used:

$$\beta = 10.0, \qquad \alpha = \frac{\pi^4}{4}$$

$$\zeta = 1.0, \qquad \omega = 3.76$$

These values were previously used by Abhyankar[7] and are similar to those employed by Holmes[3] in his respective studies of the buckled beam. In the first case, the responses of both the open- and closed-loop systems were found for various forcing amplitudes. For $F_0 = 2.0$, a chaotic time history appeared in the open-loop response. Figure 3a is a plot of the magnitude of the modal amplitude q vs a nondimensional time in which one unit represents one period of the forcing function. A significantly different motion is observed for the same forcing amplitude in the closed-loop system (Fig. 4a) where the controller has rendered the response periodic.

In the second case, the controller was required to detect the onset of chaos and then activate the actuators to alter the response. The forcing amplitude was gradually increased from $F_0 = 1.65$ to $F_0 = 2.00$, which resulted in an open-loop response that progressed from periodic to chaotic as F_0 increased (Fig. 3b). With the controller monitoring the motion, however, the system response undergoes a change as the controller senses the first bifurcation from one- to two-period motion and feedbacks a signal to the actuators. After a brief transient motion, the controller maintains periodic motion, with the amplitude increasing as the load amplitude increases (Fig. 4b). For these results, variations of magnitude 0.01 were allowed in either the position or velocity values obtained from sampling the response. By increasing these bounds for detecting the onset of chaos, it was possible to allow the system to experience higher period motion before the controller activated the actuators. Finally, it was assumed that the value of α in the controller could be in error. This gave a crude indication of the sensitivity of the closed-loop system to parameter changes. The time histories in Fig. 5 represent a 10% variation in α. These results show that the control system can maintain periodic motion when the model has small inaccuracies.

Smart Beam

During the past few years, there has been a considerable amount of research activity focused on controlling vibrations in beams by the use of piezoceramic sensors and actuators coupled by an appropriate controller. Piezoceramic sensors create an electronic signal when deformed that is proportional to the rate of change of the slope of the beam. When this signal

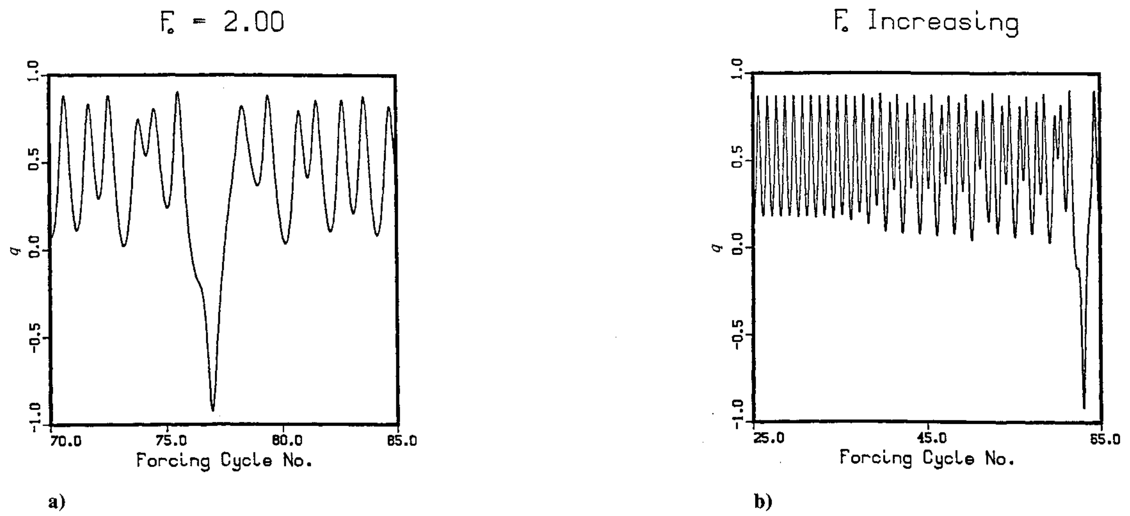

Fig. 3 Open-loop responses: a) time history for $F = 2.0$ and b) time history for F increasing from 1.65 to 2.0.

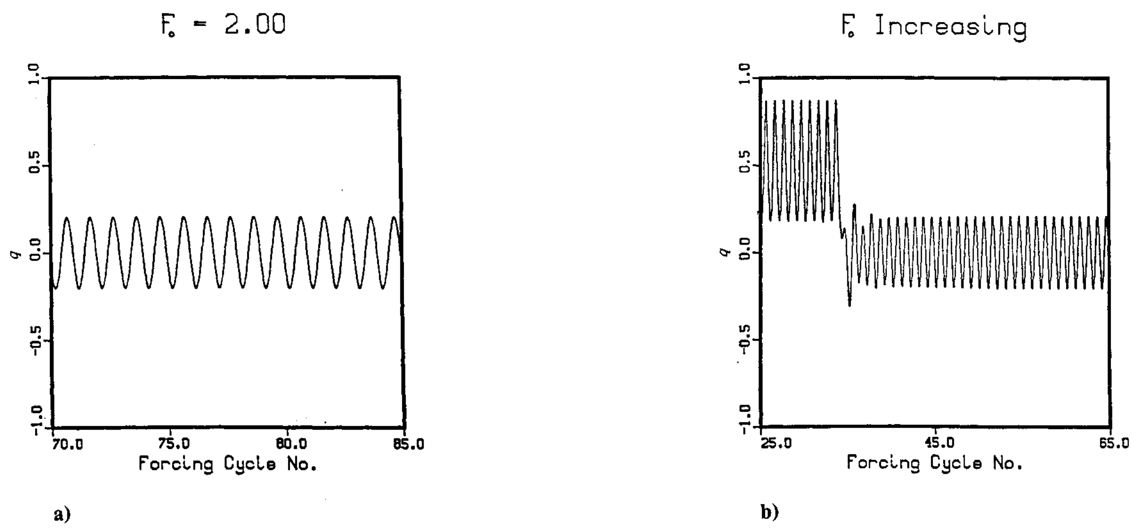

Fig. 4 Feedback linearization control: a) closed-loop time history for $F = 2.0$ and b) closed-loop time history for F increasing from 1.65 to 2.0.

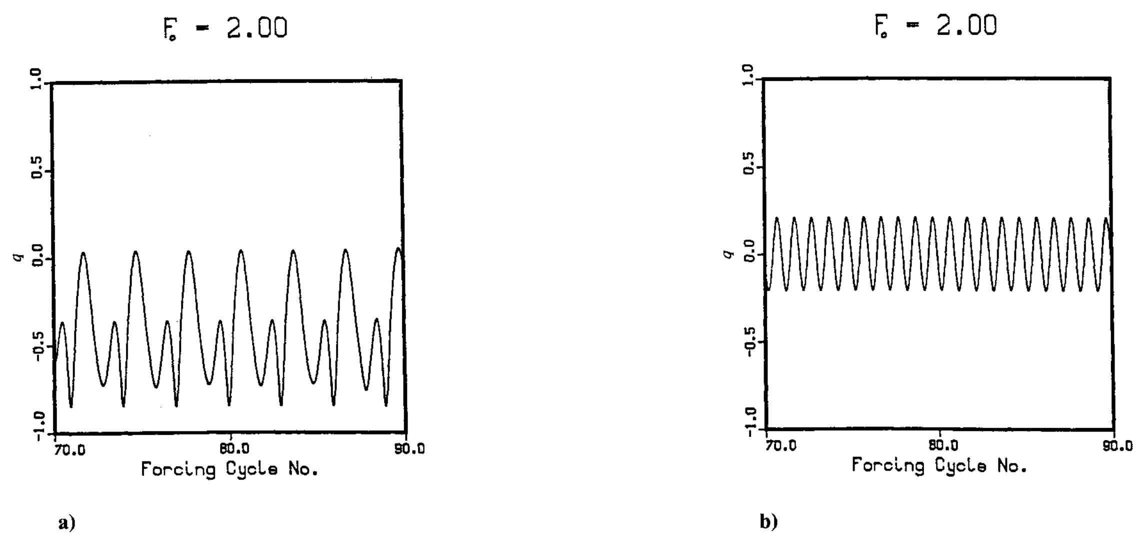

Fig. 5 Effect of parameter changes, $\Delta\alpha = 10\%$: a) open-loop time history and b) controlled system time history.

is conditioned and fed back to a piezoceramic actuator as a voltage, the actuators expand (or contract), exerting concentrated moments on the beam that are again proportional to slope rates. Thus, such a system provides a source of active damping in the controlled system. The objective in this paper is to use such active damping to eliminate chaotic vibrations and render the beam to a state of periodic vibration.

Consider then an integrated structure (Fig. 6) in which piezoceramic sensors and actuators are bonded to the buckled beam and a controller is configured to amplify and condition a signal from the sensor and feed it back to the actuator. Hanagud et al.[15] have addressed the modeling of such systems, and under assumptions such as a negligible change in beam stiffness and perfect bonds, they have found that the piezoceramic feedback control system exerts concentrated moments on the beam at the ends of the actuators. For the present study, it is assumed that the sensor and actuator run over the entire length of the beam. In this fashion, the concentrated moments will be applied at the boundaries of the beam. The magnitude of the moments is also given by Hanagud et al.[15] and has a value of

$$\hat{M}(t) = -kG\left[\dot{\phi}'(\xi=1) - \dot{\phi}'(\xi=0)\right] \tag{17}$$

where G is the gain of the controller and k a constant depending on the piezoelectric material and geometry of the beam. With these control moments, the boundary conditions of the problem are no longer homogeneous, and using the mode $\varphi = \sin \pi\xi$ in the Galerkin procedure results in a nonzero boundary residual. The effect of this boundary residual is a velocity-dependent term in the resulting ODE that reflects the damping the piezoceramic actuator provides. In this case, the model is written as

$$\ddot{q} + \zeta\dot{q} - \beta q + \alpha q^3 = F_0 \sin \omega\tau - \frac{16\pi^2 k}{EI} G\dot{q} \tag{18}$$

where β and α are the same as before. This model shares the behavior of Eq. (3), but the variable gain G allows some control of the type of response for a given forcing. An alternative φ that allows nonzero moments at the boundaries, such as

$$\varphi = \sin \pi\xi + \tfrac{1}{2}(\xi^2 - \xi)$$

could also be used, but the resulting equation has a form identical to that of Eq. (23). Only the constants are different. Thus to be consistent with the model used in the feedback linearization studies [Eqs. (4-6)], Eq. (18) will be used in this paper since it has the same constants.

Feasibility of a Smart Beam

To demonstrate the feasibility of using the foregoing smart beam, several numerical studies were completed. The beam parameters from the feedback linearization study were used, and piezoelectric constants and beam geometries from Hanagud et al.[15] utilized to obtain $k = 0.131769$. This being a feasibility study, various constant gain values were tried. As in the feedback linearization problem, the first study just analyzed the closed-loop response in comparison with the open-loop response. The open-loop system was identical to the feedback linearization study, so again chaos was observed for $F_0 = 2.0$ (Fig. 3a). The closed-loop response under the same forcing amplitude was periodic as desired for appropriate gains, with $G = 0.015$ causing single-period motion (Fig. 7a). The second study again required the controller to detect the onset of chaos and control the system under increasing forcing amplitude. The smart beam proved more effective than the feedback linearization in this study as transients were negligible and periodic behavior was maintained. Figure 7b can be compared to the open-loop results of Fig. 3b to see the effect of the smart beam controller with $G = 0.015$

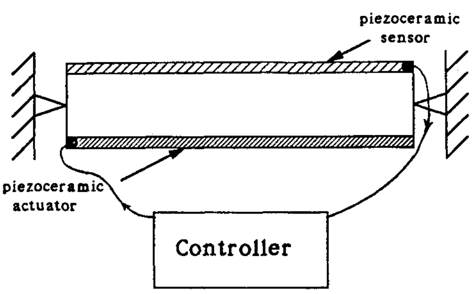

Fig. 6 Smart beam configuration.

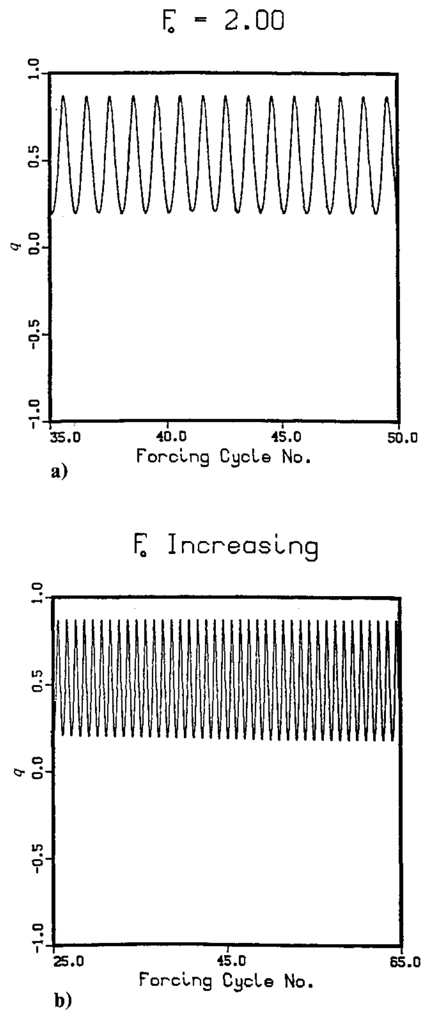

Fig. 7 Smart beam control: a) closed-loop time history for $F = 2.0$ and b) closed-loop time history for F increasing from 1.65 to 2.0.

Discussion

In numerical studies of both control systems, results demonstrate that chaos can be controlled. Feedback linearization achieved this desired goal, however, by changing the nature of the system. Without nonlinearities, the beam was forced to oscillate about its straight equilibrium position. In the smart beam, however, the nonlinear nature of the system was maintained while performing the desired task of eliminating chaotic behavior. This fact may or may not be significant in a given problem, depending on the function of the system. Additionally, the feedback linearization exhibited transients because the beam jumped toward the straight position when the actuators were activated, while the smart beam did not have any significant transients. Of course, if this were a problem, a larger gain K could be used in the feedback linearization case.

In practical applications, however, the beam equipped with piezoceramic sensors and actuators is desirable particularly because of its easy implementation and because a control system can be designed for the specified transducers, rather than designing transducers for a specified controller as in the feedback linearization case. In addition, feedback linearization cannot generally guarantee robustness, so parameter changes could also render the method ineffective, although in the present case this behavior was not observed. But in any case, the present study is merely a first step in the area of controlling chaotic vibrations. Many more studies are required, such as coupling adaptive control with the feedback linearization scheme. In addition, appropriate control theories must be developed for the smart beam technique, as the present results only form a feasibility study. However, this work does show that the control of chaotic vibrations is possible in structural dynamic systems and provides a base for further study.

Conclusions

In conclusion, this paper has demonstrated that it is a relatively simple process to control chaotic vibrations in a nonlinear beam model. Both feedback linearization and piezoceramic sensor/actuator pairs were demonstrated to be effective analytically. The so-called smart beam performed a little better in the given numerical studies by controlling the beam without changing the nonlinear nature of the response. For implementation in the lab, the smart beam is also more desirable. However, these results simply represent the initial study of controlling chaos and further investigations are required.

Acknowledgments

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